Ascending runs of sequences of geometrically distributed random variables: a probabilistic analysis

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We consider geometrically distributed random variables (RV), with the distribution GEOM(p), given by $p(i) := pq^{i-1}, q := 1-p$. Assume that we have a sequence $a_1 a_2 \dots a_n$, obtained by independent GEOM(p) RVs. An ascending run (run for short) is a maximal increasing subsequence. For example 1 1 3 5 2 4 7 1 3 3 8 has five runs, the second being 1 3 5.

The second author [**?**] presented several enumerative and asympotic results about ascending runs. In this paper, questions about ascending runs are considered from a more probabilistic point of view. In particular we try to continue the approach the first author used in [**?**]. That accounts to consider the runs either as a stochastic process or as a polyomino. In this way, the derivation of several asymptotic distributions of RVs and processes such as asymptotic Markov chain and limiting trajectories, runs number and run length distribution, hitting time to a length k run and maximum run length can be achieved.

A general point of interest is the limit $q \to 1$, as the model turns into the model of random permutations. Consequently, one obtains results that depend only on the order statistics as corollaries.

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